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Evolution of Orbit and Rotation of a Pseudo-Synchronous Binary System on the Main Sequence

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Abstract: We study the pseudo-synchronous orbital motion of a binary system on the main sequence. The equations of the pseudo-synchronous orbit are derived up to $O(e^4)$ where e is the eccentricy of the orbit. We integrate the equations to present their solutions. The theoretical results are applied to the evolution of the orbit and spin of the binary star Y Cygni, which has a current eccentricity of $e_0 = 0.142$. We tabulate our numerical results for the evolution of the orbit and spin per century. The numerical results for the semi-major axes and rotational angular velocities in the evolutional time scales of three stages (synchronization, circularization, and collapse time scale) are also tabulated. Synchronization is achieved in about 5×10^3 years followed by circularization lasting about 1×10^5 years before decaying in 2×10^5 years.

Key words: stars: binaries — stars: kinematics and dynamics — stars: individual: Y Cygni

1. Introduction

In a close detached binary system, tidal interaction couples the spin and orbital angular momenta of the component stars. The orbit and spin of a binary system evolve from a non-synchronous orbit to a synchronous one gradually under the influence of tidal friction. It is important and meaningful to study the synchronization process of the orbit and spin. The evolution of the orbit and spin is always from non-synchronization to synchronization, where the synchronized orbit is the final destination of the evolution. This is an inevitable tendency of the evolution from the younger binary stars to old binary stars.

Generally speaking, the synchronization of binary stars requires that the rotational angular velocity Ω equals the orbital angular velocity ω. However, strictly speaking, truly synchronous binary stars are found only very rarely among binary systems. What is observed instead is that each component star settles in a state of pseudo-synchronization $\Omega \neq \omega$. All binary systems appear to pass through a phase of pseudo-synchronous orbital motion. Therefore, pseudo-synchronous orbits are quite common in binary systems. Hence, it is very important to study the evolution of orbital and spin periods of binary stars.

Zahn (1977, 1978, 1989) and Zahn & Bouchet (1989) established the set of equations describing the orbital evolution of non-synchronization of binary systems. However, they did not investigate the orbit and spin of the pseudo-synchronization. Hut (1980, 1981) defines pseudo-synchronization and showed that the spin period converges to a value slightly smaller than the orbital period by an order of e^2 .

Li (2012, 2013, 2015) studied the orbit and spin synchronization of binary stars using analytical and numerical solutions, but this study does not cover the pseudo-synchronization process. We investigate the orbit and spin of pseudo-synchronization of binary stars and obtain the solution of the set of the equations of pseudo-synchronization.

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2. Definition of Pseudo-Synchronization

Zahn (1977) wrote down the equations for the orbit and spin of a non-synchronized binary system and in Zahn (1978) also revised the equations for the time rate change of eccentricity de/dt and spin rate $d(I\Omega)/dt$ of the primary star as follows.

$$\frac{1}{a}\frac{da}{dt} = -12\frac{k_2}{t_f}q(1+q)\left(\frac{R}{a}\right)^8\left[\left(1-\frac{\Omega}{\omega}\right) + e^2\left(23-\frac{27\Omega}{2\omega}\right)+O(e^4)\right] \quad (1)$$

$$\frac{1}{e}\frac{de}{dt} = -3\frac{k_2}{t_f}q(1+q)\left(\frac{R}{a}\right)^8\left[\left(18-\frac{11\Omega}{\omega}\right)+O(e^2)\right] \quad (2)$$

$$\frac{d(I\Omega)}{dt} = 6\frac{k_2}{t_f}q^2MR^2\left(\frac{R}{a}\right)^6\left[(\omega-\Omega) + e^2\left(\frac{27\omega}{2}-\frac{15\Omega}{2}\right)+O(e^4)\right]. \quad (3)$$

Here, M, R and I denote the mass, radius, and moment of inertia of the primary star and M_2 denotes the mass of the secondary star. The parameter $q = M_2/M$ is the mass ratio of the primary and secondary stars. The parameters a, e, ω denote semi-major axis, eccentricity and mean motion. We denote the apsidal motion by k_2 and the friction time by t_F introduced by Zahn &

Table 1
Orbital elements and derived parameters for Y Cygni

Y Cyg	P(d)	a($R_\odot$)	M($M_\odot$)	q	R($R_\odot$)	e	$\log L(L_\odot)$	k_2	V(km s^{-1})
Orbit elements	2.996	28.49	17.57	0.97	5.93	0.142	4.653	0.005745	146
Y Cyg	$t_{\rm F}$ (yr)	$k_2/t_{\rm F}$(yr^{-1})	MR^2/I	ω	Ω	N			
Derived values	0.1038	0.0553	8.2508	2.0965	3.5735	49.59			

P, a, M, q, R, L and e are from Simon et al. (1994) and Ibanoglu & Soyudugan (2006). Data for k_2 are from Peraiah (1965). The value of $t_{\rm F}$ is calculated using Equation (4).

Bouchet (1989)

$$t_F = \left(\frac{MR^2}{L}\right)^{1/3}. \qquad (4)$$

Here, L is the stellar luminosity.

Hut (1981) gives the definition of the pseudo-synchronization, describing the phenomenon of near synchronization of revolution and rotation around periastron, where the tidal interaction is the strongest. Pseudo-synchronism is therefore important in binary systems with substantial eccentricity. As Hut (1981) discusses, Mercury apparently achieved pseudo-synchronism with the Sun where its spin rate is near its orbital revolution near perihelion.

According to the definition by Hut (1981), $\Omega = \Omega_{ps}$ if $d\Omega/dt = 0$ in Equation (3). It can be obtained for definition of the condition of the pseudo-synchronization, from which $(\omega-\Omega)+e^2\left(\frac{27\omega}{2}-\frac{15\Omega}{2}\right) = 0$ and $6(k_2/t_f)q^2MR^2\,(R/a)^6 \neq 0$. Hence, the condition of the pseudo-synchronization can be written as

$$\frac{\Omega}{\omega} = \frac{2+27e^2}{2+15e^2}. \qquad (5)$$

This shows that the departure from strict synchronism ($\Omega = \omega$) is of order e^2.

3. Pseudo-Synchronous Equations and Their Solutions

Substituting the condition for the pseudo synchronization (5) into Equations (1) and (2), we obtain the following two equations for pseudosynchronization

$$\frac{1}{a}\frac{da}{dt} = -6K_1e^2\left(\frac{14-39e^2}{2+15e^2}\right), \qquad (6)$$

and

$$\frac{1}{e}\frac{de}{dt} = -3K_1\left(\frac{14-27e^2}{2+15e^2}\right). \qquad (7)$$

Here, the parameter K_1 is given by

$$K_1 = (k_2/t_F)q(1+q)(R/a)^8. \qquad (8)$$

With the presence of the term $O(e^4)$, Equations (6) and (7) are valid up to the order of e^3. As we will present in Section 4, we will apply our theoretical model to the binary system Y Cyg, for which the eccentricity $e = 0.142$, $e^4 = 0.0004$ is small enough. Hence Equations (6) and (7) may be neglected for the order $O(e^4)$.

From Equations (6) and (7), we have

$$\frac{da}{de} = \frac{da}{dt}\frac{dt}{de} = 2ae\left(\frac{14-39e^2}{14-27e^2}\right), \qquad (9)$$

leading to

$$\begin{aligned}\frac{da}{a} &= \left(\frac{14-39e^2}{14-27e^2}\right)d(e^2),\\ &= \frac{1}{14}[14-12e^2+O(e^4)]d(e^2). \qquad (10)\end{aligned}$$

Integrating this equation, we have

$$\ln\left(\frac{a}{a_0}\right) = \int_{e_0}^{e}\left(1-\frac{6}{7}e^2\right)d(e^2) \qquad (11)$$

which leads to

$$\ln(a/a_0) = (e^2-e_0^2) - \frac{6}{7}(e^4-e_0^4) + O(e^6). \qquad (12)$$

Neglecting terms of order e^4, we obtain

$$a = a_0\exp(e^2-e_0^2). \qquad (13)$$

Next, we solve Equation (7) in order to obtain variation of the eccentricity with time. Writing Equation (7) in the following form

$$\frac{1}{e}\frac{de}{dt} = -3K_2\left(\frac{14-27e^2}{2+15e^2}\right)a^{-8}, \qquad (14)$$

where the parameter K_2 is defined by

$$K_2 = (k_2/t_F)q(1+q)R^8. \qquad (15)$$

Substituting for Equation (10) into Equation (11), it can be written as

$$\frac{1}{e}\frac{de}{dt} = -\frac{3k_2}{a_0^8}\left(\frac{14-27e^2}{2+15e^2}\right)e^{-8(e^2-e_0^2)} \qquad (16)$$

which can be rearraged to

$$\left(\frac{2+15e^2}{14-27e^2}\right)e^{8e^2}\frac{de}{e} = -3K_2a_0^8e^{8e_0^2}dt. \qquad (17)$$

Neglecting terms of order e^4 and higher, one can obtain

$$\left[\left(\frac{31}{28}+\frac{27}{196}\right)+\frac{1}{14e^2}\right]de^2 = -3K_2a_0^{-8}\exp(8e_0^2)dt. \qquad (18)$$

Table 2
Numerical results for the change of the orbital elements and spin of Y Cyg during 100 years

Orbital Elements	a/a_0	$a(R_\odot)$	$\delta a(R_\odot)$	e/e_0	e	δe	$\delta\Omega$
Y Cyg	0.9999	28.487	−0.0004	0.9996	0.1419	−0.0001	0.00005

Integration from t_0 and e_0 to t and e yields

$$\left(\frac{31}{28}+\frac{27}{196}\right)(e^2-e_0^2)+\frac{1}{14e^2}N(e)(e^2-e_0^2)de^2 = -3K_2a_0^{-8}\exp(8e_0^2)dt, \tag{19}$$

where the expression $N(e)$ is

$$N=(\ln e^2-\ln e_0^2)/(e^2-e_0^2). \tag{20}$$

Substitution of the expression of K_2 leads to

$$e^2-e_0^2=-\frac{3k_2}{t_f}\frac{q(1+q)(R/a_0)^8\exp(8e_0^2)(t-t_0)}{\frac{N}{7}+\frac{31}{28}+\frac{27}{196}}, \tag{21}$$

from which

$$e=e_0\left[1-\frac{3k_2}{t_f}\frac{q(1+q)(R/a_0)^8\exp(8e_0^2)(t-t_0)}{\left(\frac{N}{7}+\frac{31}{28}+\frac{27}{196}\right)e_0^2}\right]^{1/2}, \tag{22}$$

Substition into Equation (13) yields

$$a=a_0\exp\left[-\frac{3k_2}{t_f}\frac{q(1+q)(R/a_0)^8\exp(8e_0^2)(t-t_0)}{\frac{N}{7}+\frac{31}{28}+\frac{27}{196}}\right]. \tag{23}$$

From Equation (5) with $e=e_0$ we may write

$$\delta\Omega=\left(\frac{2+27e_0^2}{2+15e_0^2}\right)\delta\omega. \tag{24}$$

Combining it with the Kepler's third law $\delta\omega = -\frac{3}{2}\left(\frac{\omega}{a}\right)da$, we obtain

$$\delta\Omega=-\frac{3}{2}\left(\frac{2+27e_0^2}{2+15e_0^2}\right)\left(\frac{\omega}{a}\right)\delta a. \tag{25}$$

Zahn (1977) defined the circulatization time $t_{\rm cir}$ by means of his non-revised Equation (2) for de/dt as

$$t_{\rm cir}^{-1}=-e^{-1}\frac{de}{dt}=\frac{63}{4}\frac{k_2}{t_{\rm F}}q(1+q)(R/a)^8\ {\rm yr}^{-1} \tag{26}$$

assuming that $\Omega=\omega$. However, in this work, we use the revised equation for de/dt with $\Omega=\omega$, in which case the circularization time should be written as

$$t_{\rm cir}^{-1}=21\frac{k_2}{t_{\rm F}}q(1+q)(R/a)^8\ {\rm yr}^{-1}. \tag{27}$$

According to Equation (3), the synchronization time $t_{\rm syn}$ becomes

$$\frac{1}{t_{\rm syn}}=-\frac{1}{\Omega-\omega}\frac{d\Omega}{dt}=6\frac{k_2}{t_{\rm F}}q^2\left(\frac{MR^2}{I}\right)(R/a)^6\ {\rm yr}^{-1} \tag{28}$$

In turn, the orbital decay time $t_{\rm decay}$ or the collapse time of the system can be written as

$$\frac{1}{t_{\rm decay}}=-\frac{1}{a}\frac{da}{dt}=12\frac{k_2}{t_{\rm F}}q(1+q)(R/a)^8 \tag{29}$$

4. Pseudo-Synchronization of Orbit and Spin in Y Cygni

We choose the binary system Y Cygni as our exemplary model for pseudo-synchronization. The spectral type of Y Cyg is traditionally given as B0 V in the literature, but has been found to be earlier, O9.8 V (Simon et al. 1994). Y Cyg is an eclipsing binary and is also very well known for its apsidal motion. According to Giuricin (1984), the rotational velocity $V=146$ km s^{-1} and the estimated synchronized velocity and $V_{\rm k}=88$ km s^{-1} for the binary star Y Cyg, which shows that this binary system is not a synchronous binary system. The orbital elements and the deduced parameters are listed in Table 1.

Noting that $e\simeq e_0$, we use l'Hospital's rule to approximate Equation (20) by

$$N(e)=\frac{\ln e^2-\ln e_0^2}{e^2-e_0^2}\simeq\frac{1}{e_0^2}. \tag{30}$$

For Y Cyg, we have $e_0=0.142$, which, in turn, leads to $N\simeq e_0^{-2}=49.6$. The use of l'Hospital's rule is justified because the variation δe is very small.

Substituting these values for data of q,e,R,N while taking $t-t_0=100$ yr into Equations (16)–(18) and (19)–(20) and (22), we obtain the change of the orbit and spin per century for Y Cyg, which is summarized in Table 2.

We estimate the changes of the orbital semi-major axis and spin of Y Cyg when the binary system achieves synchronization, circularization, and collapse. Substituting these values of data in Table 1 into Equations 28,27 and 29, we obtain the values of $t_{\rm cir}$, $t_{\rm syn}$ and $t_{\rm decay}$.

$$t_{\rm syn}=4.774\times10^5\ {\rm yr},$$
$$t_{\rm cir}=1.279\times10^5\ {\rm yr}$$
$$t_{\rm decay}=2.239\times10^5\ {\rm yr} \tag{31}$$

Substituting Equation (31) into Equations (23), (24), and (25), we obtain the values for the three cases of evolutional stages of pseudo-synchronization of Y Cyg listed in Table 3.

5. Conclusion

It can be seen from Table 2 that the orbital parameters and spin exhibit very slow changes for 100 years due to the action of the tidal friction for Y Cyg. We may also find from Table 3 that Y Cyg achieves or enters the synchronization stage firstly, and then, circularization stage and finally enter the orbital collapse stage.

From Table 3, we may assert that in the system of Y Cyg the orbital semi-major axis, a, decreases with age monotonically and the spin rate, Ω, increases with age

Table 3
Evolution of semi-major axis and spin of Y Cyg

Stage	t (yr)	a/a_0	a ($R_\odot$)	δa ($R_\odot$)	$\delta\Omega$ (rad/d)
A	4.774×10^3	0.9991	28.4643	-0.0256	0.0031
B	1.279×10^5	0.9811	27.9515	-0.05384	0.0065
C	2.239×10^5	0.9669	26.852	-0.9430	0.1998

Numerical results for the three evolutional stages of the orbital semi-major axis and spin of Y Cyg. The stages A, B and C stand for synchronization, circularization and collapse, respectively.

continuously in each stage. In the synchronous stage the decrease is large for $\delta a_{\rm syn}$, and the increase is large for $\delta\Omega_{\rm syn}$. In the circularization stage, we find that $\delta a_{\rm cir} < \delta a_{\rm syn}$ and $\delta\Omega_{\rm syn} > \delta\Omega_{\rm cir}$. However, in the orbital collapse stage, $\delta a < \delta a_{\rm cir} < \delta a_{\rm syn}$ and $\delta\Omega > \delta\Omega_{\rm cir} > \delta\Omega_{\rm syn}$.

It should be noted that Equations (1) and (2) are valid up to second order of eccentricity e and that higher order terms $O(e^4)$ are neglected. Hence these equations should be used for binary systems with a small eccentricity, and may be inadequete to those with a large eccentricity. For the eccentricity $e = 0.142$ of the eclisping binary system Y Cyg, we have $e^4 = 0.0004$, which is sufficiently small validating our use of Equations (1) and (2).

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