Gaia Parallax Zero Point From RR Lyrae Stars

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Abstract: Like Hipparcos, Gaia is designed to give absolute parallaxes, independent of any astrophysical reference system. And indeed, Gaia's internal zero-point error for parallaxes is likely to be smaller than any individual parallax error. Nevertheless, due in part to mechanical issues of unknown origin, there are many astrophysical questions for which the parallax zero-point error $\sigma(\pi_0)$ will be the fundamentally limiting constraint. These include the distance to the Large Magellanic Cloud and the Galactic Center. We show that by using the photometric parallax estimates for RR Lyrae stars (RRL) within 8kpc, via the ultra-precise infrared period-luminosity relation, one can independently determine a hyper-precise value for π_0. Despite their paucity relative to bright quasars, we show that RRL are competitive due to their order-of-magnitude improved parallax precision for each individual object relative to bright quasars. We show that this method is mathematically robust and well-approximated by analytic formulae over a wide range of relevant distances.

Key words: astrometry

1. Introduction

Gaia will obtain astrometry for $> 10^9$ stars, with parallax precisions down to $\sigma(\pi) \lesssim 6\mu$as for the brightest stars, $V \lesssim 12$. In contrast to traditional pre-Hipparcos astrometry, Gaia is designed to measure so-called "absolute parallaxes". However, since nothing in nature is truly "absolute", it behooves us to specify more precisely exactly what Gaia will measure.

In traditional narrow angle astrometry, one measures the parallactic motion of some target star relative to a set of reference stars, and from this measures the "relative parallax" $\pi_{\rm rel} = \pi_{\rm target} - \pi_{\rm reference}$, where the last quantity is the mean parallax of the reference stars. One then estimates the distances of the reference stars, and hence $\pi_{\rm reference}$, by some non-astrometric method, usually photometric. If, for example, the reference stars are five times farther than the target star, and if their distances can be estimated to, say 30% precision, and assuming $N = 4$ reference stars, then the contribution of the error due to the reference frame is only $\sigma(\pi_{\rm reference})/\pi_{\rm target} = 30\%/5/\sqrt{4} = 3\%$, which may well be lower than the contribution from the astrometric precision of the $\pi_{\rm rel}$ measurement. In a variant of this approach, one might use external quasars or galaxies as the reference frame, in which case $\pi_{\rm reference} = 0$ to a precision adequate for most purposes.

By contrast Hipparcos used wide-angle astrometry, which does not require any external reference frame for parallaxes (although it does for proper motions). To understand the basic principle of this approach, consider two telescopes that are rigidly separated by 90°.

Let the first of these telescopes make two measurements of a star in the ecliptic, six months apart, both times at quadrature. That is, both of these measurements will suffer maximal parallactic deflection, but in opposite directions. Now let the second telescope measure the positions of a second star, also in the ecliptic at the same epochs. Since this second star is, by construction, aligned perfectly with the Sun, it will not suffer any parallactic deflection. Hence the relative change in position of these two stars directly gives the absolute parallax of the first. Now, of course, one of these two measurements of the second star could not be made in practice because it would lie directly behind the Sun. However, the point is that by simultaneously observing stars that are affected by parallax by substantially different (and easily calculable) amounts, one can extract the absolute parallax.

For this method to work to a given specified precision, the "basic angle" between the two telescopes must remain fixed to the same precision. Or rather, any changes in the basic angle must be understood to this specified level of precision. If the basic-angle oscillations have power on timescales shorter than the rotation period of the two telescopes, then the amplitude of these oscillations can be derived (and so corrected) from the observations themselves. However, oscillations at the rotation period are indistinguishable from a zero-point offset of all parallaxes that are being measured. Uncertainty about this amplitude is therefore equivalent to introducing a "$\pi_{\rm reference}$" term, as in narrow-angle astrometry.

For reasons that are not presently understood, the actual amplitude of these oscillations is about 1 mas,

which is orders of magnitude higher than expected from the original design, and also orders of magnitude higher than the parallax precision of the best measurements. Happily, the great majority of this oscillation can be measured from engineering data, but an ultra-precise estimate of the Gaia system parallax zero point, π_0, will require external calibration.

It may well be that for most applications a precision determination of π_0 is irrelevant. However, it is easy to imagine applications for which this is important. For example, the parallax of the Large Magellanic Cloud (LMC) is presently estimated to be $\pi_{\mathrm{LMC}} = 20\,\mu$as (for a comparison of measurements see de Grijs et al. 2014). Consider measurements of 10,000 LMC stars at $V = 16$, each with precision $\sigma(\pi) = 40\,\mu$as. Each measurement by itself would be "useless", having a 200% error. Nevertheless, the combination of all of them would have an error $\sigma(\pi_{\mathrm{LMC}}) = 0.4\,\mu$as, i.e., a 2% error. However, if the zero-point error $\sigma(\pi_0) \sim 2\,\mu$as, then this LMC distance measurement would be degraded by a factor 5.

One method to measure π_0 is from quasars. There is about one such object per square degree to $V_0 = 18$ (e.g., Hewett et al. 2001). For the $\sim 3/4$ of these that are relatively unextincted, the Gaia precision[1] is anticipated to be $\sigma(\pi) \sim 140\,\mu$as. Since these are each known a priori to have zero parallax (or rather $\pi \ll 1\,\mu$as), the 30,000 that lie over 3π sterradians can be combined to yield $\sigma(\pi_0) \sim 0.8\,\mu$as. There are ~ 3 times more quasars $(18 < V_0 < 19)$ than $V_0 < 18$, but each contributes substantially less information. Including all quasars, we estimate $\sigma(\pi_0) \sim 0.6\,\mu$as from this technique.

This estimate then sets the benchmark for other techniques. If these other methods can achieve a similar or better precision, then they can serve as an independent check on the quasars and improve the overall measurement of π_0.

2. RR Lyrae Star Based Zero Point: Naive "Circular" Argument

At infrared wavelengths, RR Lyrae stars obey a period-luminosity (PL) relation

$$L_\lambda = L_{0,\lambda}(P/P_0)^\beta \qquad (1)$$

where P_0 is chosen to be near the mean period of the sample (Longmore et al. 1986, Longmore et al. 1990). There is some scatter around this relation, which is usually expressed in magnitudes $\sigma(M_\lambda) = (5/\ln 10)\langle(\delta L)^2\rangle^{1/2}/L$, but which we will express for convenience in terms of the error in inferred distance

$$\epsilon = \frac{\langle(\delta L)^2\rangle^{1/2}}{L} \qquad (2)$$

[1] The Gaia site http://www.cosmos.esa.int/web/gaia/science-performance gives $\sigma(\pi)$ = $(-1.631 + 680.766z + 32.732z2)1/2[0.986 + (1 - 0.986)(V - I)$ where $z = \min(10^{0.4(G-15)}, 10^{-1.2})$

At present, ϵ is not known because it appears to be below the precision of the best RR Lyrae parallax measurements made to date (e.g., Benedict et al. 2011; Madore et al. 2013; Dambis et al. 2014; Braga et al. 2015). It may plausibly be $\epsilon \sim 0.01$ or even less depending on wavelength (although see theoretical estimates from Bono et al. 2001). This scatter will be easily probed by Gaia. RR Lyrae stars with distance $D < 2\,$kpc will have parallax errors $\sigma(\pi) \sim 6\,\mu$as and therefore fractional distance errors $\sigma(\pi)/\pi \sim 0.6\%(D/1\,\mathrm{kpc})^{-1}$. Thus, it will be quite noticeable if the nearby RR Lyrae stars show intrinsic luminosity scatter such that $\epsilon > 0.01$.

The basic approach then is to measure L_0 using the relatively nearby stars $D \lesssim 2\,$kpc (see recent overview by Beaton et al. 2016). Because their parallaxes are so much larger than any possible zero point error, the latter can to first approximation be ignored, and the very high precision parallax measurements can then be used to measure L_0 (as well as σ). Then one can apply this knowledge to much more distant RR Lyrae stars, e.g., at $D \sim 5\,$kpc. While the parallaxes of these stars $(\pi \sim 200\,\mu$as) are also much larger than any possible π_0, the error in the individual distances due to scatter in the PL relation is only $\epsilon\pi \sim 2\,\mu$as. Not only is this now of order the plausible values of π_0, more to the point it is much smaller than the Gaia measurement error for these stars, $\sigma(\pi) \sim 15\,\mu$as. Since the uncertainty in the photometric parallax estimate is much smaller than the parallax measurement error, it basically does not contribute. Hence, each parallax measurement of such relatively distant RR Lyrae stars constitutes an independent estimate of π_0 with error $15\,\mu$as per star. Thus, even though there are many fewer RR Lyrae stars than quasars, they can be competitive because of much smaller errors for each measurement.

Then, with π_0 measured, one can go back and improve the determination of L_0 by properly accounting for this zero-point offset. The last step may appear circular, but we will see that each element of this description, including the naively "circular" argument, maps directly onto a rigorous statistical approach.

3. Mathematical Description

Strictly speaking we should simultaneously fit for four parameters, L_0, β, ϵ, and π_0. However, β and ϵ are essentially uncorrelated from the other parameters. In the interests of focusing on the main determinants of the problem, we will take β and ϵ as given.

We can then write the relation between observed and modeled parallaxes

$$\pi_{\mathrm{obs,k}} = \pi_0 + A\pi_{\mathrm{fid},k} \pm \sigma_k; \quad \pi_{\mathrm{fid}} \equiv \sqrt{\frac{4\pi F_{\mathrm{dered},k}}{L_{0,\mathrm{fid}}(P/P_0)^\beta}} \qquad (3)$$

where $F_{\mathrm{dered},k}$ is the dereddened observed flux of the kth star in the appropriate infrared band, and $L_{0,\mathrm{fid}}$ is the initial guess for L_0 (which will then be corrected by measuring A). The error σ_k is the quadrature sum

of two contributions. The first is from the scatter in the PL relation, namely $\epsilon\pi$. The second is the measurement error. For now we will assume that all the stars are in the photon limit[2] and that therefore this error is inversely proportional to the square root of the flux in the Gaia bands. Since RR Lyrae luminosities are roughly independent of period in optical bands, this implies (if we restrict attention to relatively unextincted stars), that the error is inversely proportional to the flux. Assuming that $M_G = 0.6$ in the Gaia band, and adopting the anticipated Gaia precision in the photon limit, one then finds

$$\sigma^2(\pi) = (\epsilon\pi)^2 + \left(\frac{\kappa}{\pi}\right)^2 \qquad \kappa = (57\,\mu\text{as})^2 \qquad (4)$$

We then follow the standard procedure of constructing a Fisher matrix and approximating it as an integral (e.g., Gould 1995). First, one forms the inverse covariance matrix of the two parameters (π_0, A), which are labeled "0" and "1", respectively

$$B_{ij} = \sum_k \frac{\pi_k^{i+j}}{\epsilon^2 \pi_k^2 + \kappa^2/\pi_k^2} = \frac{1}{\epsilon^2}\sum_k \frac{\pi_k^{i+j-2}}{1 + (\kappa/\epsilon)^2/\pi_k^4} \quad (5)$$

Switching variables to distance $r = \text{AU}/\pi$ and taking the sum to an integral, we obtain

$$B_{ij} = \frac{1}{\epsilon^2}\sum_k \frac{(r_k/\text{AU})^{2-i-j}}{1 + (r_k/D_*)^4} \qquad (6)$$

$$B_{ij} \to \frac{3\pi n(\text{AU})^{i+j-2}}{\epsilon^2}\int_0^{r_{\max}} dr \, \frac{r^{4-i-j}}{1 + (r/D_*)^4} \qquad (7)$$

where we have assumed a uniform density n and that the RR Lyrae stars can be effectively incorporated only over 3π sterradians. Here

$$D_* \equiv \text{AU}\sqrt{\frac{\epsilon}{\kappa}}. \qquad (8)$$

Substituting $x = r/D_*$ yields

$$B_{ij} = \frac{3\pi n D_*^3}{\epsilon\kappa}\left(\frac{\kappa}{\epsilon}\right)^{\frac{i+j}{2}} b_{ij}(x_{\max}), \quad b_{ij}(x) = \int_0^x dy\,\frac{y^{4-i-j}}{1+y^4} \qquad (9)$$

Unfortunately, only the off-diagonal terms of b_{ij} can be evaluated in closed form, $b_{01}(x) = \ln(1+x^4)^{1/4}$. However, for $x \gtrsim 2.5$, b very quickly approaches its asymptotic limit[3]

$$b_{ij}(x) \to \begin{pmatrix} x-w & \ln x \\ \ln x & w-x^{-1} \end{pmatrix}, \quad w \equiv (1/4)!(-1/4)! \qquad (10)$$

The constant $w = (1/4)!(-1/4)!$ is obviously[4] just slightly larger than unity, $w \simeq 1.111$. The naive argument given in Section 2 maps directly onto Equations (9) and (10). The prefactor $3\pi n D_*^3 = 3N_*$ is (3 times) the number of RR Lyrae stars within the radius D_* at which the astrometric and PL-relation errors are equal. The information content about π_0 is equivalent to a naive integral outside this radius, $b_{11} \simeq N_*(x-1)/\epsilon^2$. The reason that the Gaia precision constant κ does not explicitly enter this formula is that the volume element (r^2) exactly cancels the distance dependence of the inverse square of the errors $(\pi/\kappa)^2$. Hence the amplitude of this essentially constant integral is set at D_* where $\kappa/\pi = \epsilon\pi$.

The information content about the PL relation is equivalent to a naive integral within most of the interior volume $b_{00} \simeq N_*(1 - 1/x)/\epsilon\kappa$.

Finally, the formal mathematical quantification of the "circular argument" given in Section 2 is the correlation coefficient ρ

$$\rho(x) = -\frac{\ln x}{\sqrt{(x-w)(w-x^{-1})}} \qquad (11)$$

For modest values of $x_{\max}$, ρ is quite large. For example, $\rho(2.5, 3, 4) = -(0.92, 0.91, 0.88)$. These high values degrade the naive information content about π_0 by

$$[\sigma(\pi_0)]^2 = C_{00} = \frac{\epsilon^2}{3N_*}\frac{b_{00}^{-1}}{1-\rho^2} \qquad (12)$$

where $C \equiv B^{-1}$ is the covariance matrix and $b_{00} \simeq x-w$

Before applying these equations to the problem of measuring π_0, we must first account for the fact that Gaia precisions do not further improve as the source gets brighter than G=12. For RR Lyrae stars, this corresponds to distance $D_{\min} = 1.9\,\text{kpc}$, and so to

$$x_{\min} = D_{\min}/D_* = 1.08\left(\frac{\epsilon}{0.01}\right)^{-1/2} \qquad (13)$$

Then the formula for the inverse covariance matrix B remains valid provided one substitutes

$$b_{ij} \to b_{ij} - \Delta b_{ij} \qquad (14)$$

where

$$\Delta b_{ij} = \int_0^{x_{\min}} dy\,\frac{y^{4-i-j}}{1+y^4} - \frac{y^{4-i-j}}{1+y^2 x_{\min}^2} \qquad (15)$$

In the relevant range of $x_{\min}$, this adjustment is quite small and below the level of the errors made by various other approximations in this treatment. For example $\Delta b_{ij}(x_{\min}=1) = (0.014, 0.020, 0.020, 0.028)$.
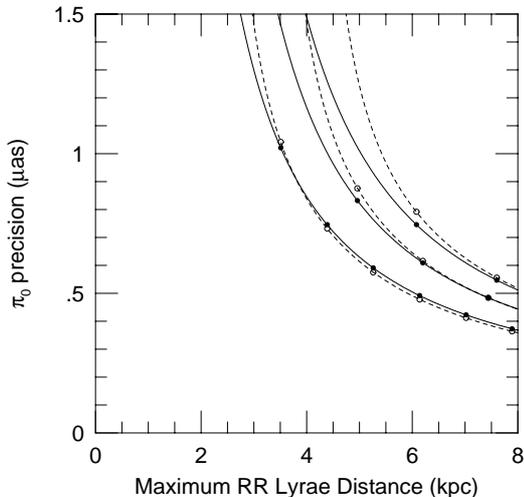
4. Numerical Estimates

To make numerical estimates of the precision that can be achieved, we first estimate $n_0 = 5.8\,\text{kpc}^{-3}$ based

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[2] A discussion of these errors is given in Gould, Kollmeier & Sesar 2016.

[3] This is because the next terms in the expansion are $+x^{i+j-5}/(5-i-j)$, i.e., 4 powers of x below the leading term (plus an additional factor of a few).

[4] because the factorial function is logarithmically convex and $0! = 1$

Figure 1. Precision of Gaia estimate of $\sigma(\pi_0)$ based on RR Lyrae star method, assuming all RR Lyrae stars are incorporated out to a maximum distance indicated on the abscissa. Solid lines show result of numerical integration assuming intrinsic distance scatter from the PL relation of $\epsilon = (0.01, 0.02, 0.03)$ (bottom to top), while dashed lines show the analytic approximation given by Equation (10). Open and filled circles are at $x = (2.0, 2.5, 3.0 \ldots)$, showing that the approximation becomes essentially exact for $x \geq 2.5$

on Hipparcos RR Lyrae stars that are $V < 11$ and that satisfy the Layden et al. (1996) "Halo-3" criteria (which were also adopted by Popowski & Gould 1998 and Gould & Popowski 1998). The restrictive magnitude limit is to ensure completeness. Of course, the so-called "thick disk" RR Lyrae stars that do not satisfy the "Halo-3" criteria will also contribute to the determination. They are nevertheless excluded to be conservative because their flattened three-space distribution implies that they will contribute much less leverage at large distances compared to halo RR Lyrae stars.

Figure 1 shows the estimated precision $\sigma(\pi_0)$ that can be achieved assuming that the RR Lyrae distance scatter in a particular IR band is $\epsilon = 0.01, 0.02,$ or 0.03, and as a function of the upper distance limit $r_{\max}$ to which are RR Lyrae measurements are essentially complete. At, for example, $r_{\max} = 5\,\mathrm{kpc}$, these precisions are $\sigma(\pi_0) = (0.63, 0.82, 1.01)\,\mu\mathrm{as}$ for $\epsilon = (0.01, 0.02, 0.03)$. This shows that the RR Lyrae star method is comparable to the quasar method.

The solid lines in Figure 1 show the results of numerical integration, i.e., using Equations (9), (14), and (15), while the dashed lines show the results of using the analytic approximation in Equation (10). The solid and open circles on these curves denote the evaluations at $x = (2.0, 2.5, 3.0 \ldots)$. As predicted analytically in the text above, the approximations are essentially perfect for $x \geq 2.5$.

Of course, RR Lyrae stars are not distributed uniformly around the Sun, but the formalism developed here only requires that this be true averaged over shells. Even this assumption is not strictly valid, but remains approximately valid for $r < 8\,\mathrm{kpc}$, since the declining density toward the Galactic anti-center is compensated by the increasing density toward the Galactic center. However, the approximation becomes completely invalid for $r \gtrsim 8\,\mathrm{kpc}$ since at that point the density is declining in all directions. Hence, the fundamental limits of the method are illustrated by the abscissa cut-off in Figure 1.

5. RR LYRAE DISTANCE SCALE

It is also of interest to estimate how well the zero point of the PL relation can be determined. From algebraic manipulation of Equations (9) and (10), this is related to $\sigma(\pi_0)$ by

$$\frac{\sigma(A)}{\sigma(\pi_0)} = \sqrt{\frac{\epsilon}{\kappa} \frac{x - w}{w - x^{-1}}} = \frac{D_*}{\mathrm{AU}} \sqrt{\frac{x - w}{w - x^{-1}}} \qquad (16)$$

Since, $\sigma(\pi_0) \sim \mathcal{O}(\mu\mathrm{as})$, while $\mathrm{AU}/D_* \sim \mathcal{O}(\mathrm{mas})$, this implies that the zero point of the RR Lyrae PL relation can be measured with precision of order 10^{-3}. From RR Lyrae itself (and 3 other RRab stars), the absolute zero point is known to approximately 5% (Benedict et al. 2011) using trigonometric parallaxes from HST. For RRc variables, the most precise values of the zero point come from the trigonometric parallax of RZ Cep (Benedict et al. 2011) and the statistical parallax analysis from the CARRS survey (Kollmeier et al. 2013), although these values are in marginal tension. As demonstrated above, *Gaia* precision will be dramatically superior.

To date, the only observables on which there has been a measurable dependence of RR Lyrae luminosity are period and metal abundance. The availablity of precision GAIA parallaxes may allow to test for additional observable parameters. One simple test (suggested by the referee) would be to measure the gradient of the luminosity (after fitting out the dependence on period and luminosity) with Galactocentric distance. Since it is known that there are at least two halo populations with different scale lengths, such a test could probe the presence of some systematic difference between these populations. If such a signature were found, it could then guide a more detailed search for observables that are preferentially in one of these populations over the other.

While it has been historically difficult to measure distances to individual objects (Bessel 1838) in the modern era precision wide-field astrometry has proved challenging. Indeed, the historic Hipparcos mission measurements opened up substantial debate on the distance to the Pleiades as the then-new astrometric determination disagreed with traditional values. At that time, it was suggested that correlated errors could be to blame (Pinsonneault et al., 1998; Soderblom et al., 1998). The Gaia collaboration has now published a

new astrometric value consistent with the historic Pleiades distance (Brown et al. 2016), but the controversy highlights the intrinsic difficulty of these measurements and the great utility of having multiple independent distance-scale crosschecks.

6. Conclusion

The Gaia mission data promises to transform our understanding of the Milky Way. In this work, we have shown that exploiting photometric parallax estimates for RRL within 8 kpc in conjunction with the precise IR P-L relation for these objects, one can measure the absolute parallax zero-point $\sigma(\pi_0)$ to precision of less than $0.5(\mu as)$. Not only is this extremely precise, but it is also comparable to, and completely independent of, measurements of this quantity from quasars. We further show that once this is determined, one can refine the precision of the IR P-L zero point well beyond what is possible from photometric measurements alone. We anticipate this independent method will be of immediate use to the astronomical community.

Acknowledgments

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